Partially Balanced Incomplete Block Designs (PBIBD).

A set of v treatments distributed in b blocks each containing k distinct treatments $(k < v)$, is said to form a P.B.I.B.D with m-associate classes when

1) every treatment occurs in r blocks.
2) two treatments occur together in $\lambda_1, \lambda_2, \dots,$ or λ_m blocks.
3). given a treatments $\oplus$ each of n_i treatments occurs with $\oplus$ in λ_i blocks $(i = 1, 2, \dots, m)$ so that $\sum_i n_i = v-1$ and $\sum_i n_i \lambda_i = r(k-1)$.

PBIBD(2) Association Schemes:

The following are the five categories according to the association schemes of the designs

1) Group Divisible (G.D).
2) Simple
3) Triangular
4) Latin Square Type
5). Cyclic.

1) Group Divisible (G.D).

In this category the $v = mn$ treatments are divided into m groups of n each such that any two treatments of the same group are first associates and two treatments from different groups are second associates. The association scheme can be displayed by arranging the treatment numbers in a rectangular arrangement of m rows and n columns where each row of n treatments gives a group. Evidently,

$$n_1 = n-1, \quad n_2 = n(m-1)$$

The second parameters are

$$p_{jk}^{1} = \begin{pmatrix} n-2 & 0 \\ 0 & n(m-1) \end{pmatrix}$$

$$p_{jk}^{2} = \begin{pmatrix} 0 & n-1 \\ n-1 & n(m-2) \end{pmatrix}$$

2) Simple PBIBD

A PBIB design with two associate classes is said to be simple if either

(i) $\lambda_1 \neq 0$, $\lambda_2 = 0$ (or)

(ii) $\lambda_1 = 0$, $\lambda_2 \neq 0$.

It may happen that a design of the simple category may belong to some other category as well.

3) <u>Triangular PBIB Design</u>

A PBIB design with two associates classes is said to be triangular if the number of treatments $v = n(n-1)/2$ and the association schemes is an array of n rows and n columns such that

(i) the position in the principal diagonal of the scheme (upper left to lower right) are left blank.

(ii) the $n(n-1)/2$ positions above the principal diagonal are filled by the treatments numbers $1, 2, \ldots, n(n-1)/2$,

(iii) the $n(n-1)/2$ positions below the diagonal are so filled that the array is symmetrical about the principal diagonal, and

(iv) for any treatment i the first associates are exactly those treatments which lie in the same row as i.

Here $n_1 = 2n-4$, $n_2 = (n-2)(n-3)/2$.

$$p^1_{ij} = \begin{pmatrix} n-2 & n-3 \\ n-3 & (n-3)(n-4)/2 \end{pmatrix}$$

$$p^2_{ij} = \begin{pmatrix} 4 & 2n-8 \\ 2n-8 & (n-4)(n-5)/2 \end{pmatrix}.$$

4) Latin Square Type of PBIB Design

Let the square array of n rows and n columns be formed with n^2 treatment numbers from 1 to n^2, so that two treatments are first associates if they occur in the same row or the same column of the array and second associate otherwise. A design with the above array as association scheme is said to belong to the sub type L_2. A design belonging to sub-type L_3 is also defined. In these designs it is possible to form a square array of n^2 treatments numbers from 1 to n^2 and to impose a latin square with n letters on this array, so that any two treatments are first associates if they occur in the same row or column of the array or correspond to the same letter of the latin square and are second associates otherwise.

In these designs,

$$n_1 = L(n-1), \quad n_2 = (n-1)(n-L+1)$$

$$p_{ij}^1 = \begin{pmatrix} L^2 - 3L + n & (L-1)(n-L+1) \\ (L-1)(n-L+1) & (n-L)(n-L+1) \end{pmatrix}$$

$$p_{ij}^2 = \begin{pmatrix} L(L-1) & L(n-1) \\ L(n-L) & (n-L)^2 + (L-2) \end{pmatrix}$$

where $L = 2$ for sub-type L_2 and $L = 3$ for sub-type L_3.

5). <u>Cyclic PBIBD</u>

A non-group divisible PBIB design is called cyclic if the set of first associates of the treatment numbered i is obtained by adding $i-1$ to the numbers in the set of first associates of the treatment numbered 1 and subtracting v whenever the sum exceeds v (the number of treatments).

The analysis of each category of designs has been illustrated by them. There are large numbers of methods for the construction of these designs.